Blind Separation of Two Signals by Estimation of Two Fourth-Order Cumulants

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Abstract - The problem is to recover stochastic signals from an unknown stationary linear mixture. The paper presents analytical solution for blind separation of two statistically independent signals. It requires two fourth-order input sample cumulants to be estimated, contrary to the solution given in references that requires estimation of three fourth-order input sample cross-cumulants. When real-time separation problem is considered this difference can be significant.

I. INTRODUCTION

The so-called blind signal separation problem has been attracted significant attention in last several years. The problem is consisted of separating and estimating generally multiple source signals from an array of sensors. It is assumed that the problem is described with:

$$y = Ax \tag{1}$$

where x,y $\in R^n$, and $A\in R^{nxn}$, n is number of signals and det$A\neq 0$. The only assumption made here is that components of x are mutually independent up to at least fourth order. To solve the blind signal separation problem the linear transformation matrix W must be find such that:

$$s = Wy \tag{2}$$

where s is scaled version of the original source signal vector x. It has been shown that (1)-(2) can be solved by using two approaches: neural network and higher-order (fourth-order) statistics. The neural network based solution was firstly given in [6], while in [7,8] a new algorithms were proposed that allow the extraction of extremely badly scaled signals i.e. the mixing matrix A can be ill-conditioned. The neural network approach requires the source signals x_j, j=1...n, to have even probability density function. The higher-order independence test is introduced indirectly by using nonlinear odd activation functions. The neural network solution enables that the number of signals to be separated can in general case be arbitrarily large, what is not the case with the cumulant based solution. The potential problems with neural networks arise when input signals are nonstationary. It becomes problematic to determine the values of the convergence control factors. By using cumulants the nonstationary case can be easier handled by computing estimates of the input sample cumulants all the time. The cumulant based solution is obtained by equating all three fourth-order output cross-cumulants with zero. Unfortunately, it has been shown in [3,5] that the cumulant based analytical solution is impossible to be found for the number of signals n > 2.

II. THE CUMULANT BASED SOLUTION OF THE TWO SIGNALS SEPARATION PROBLEM

The k-th order cumulant of the random variable x_j is defined as the k-th coefficient of the Taylor series expansion of the second characteristic function [1,2]:

$$K(\omega) = \ln \Phi(\omega) = \ln E\left\{e^{j\omega x^T}\right\}$$

where $\omega=[\omega_1 ... \omega_2]$, and $\Phi(\omega)$ is characteristic function of x. Let the a_{ij} and w_{ij}, (i,j=1,2) are elements of the matrices A and W in accordance with (1) and (2). It is usually assumed that the separation signals s_1 and s_2 represent the source signals x_1 and x_2 up to the scale factors. The s_1 and s_2 are considered to be separated when mutual higher order statistics is zero. In practice the fourth-order statistics is required to be zero. The third order statistics is avoiding since most of the real world signals, because of their symmetrical distribution, have the third order statistics nearly zero. The fourth-order cumulants are used instead of moments since their linearity property, [1,2], let us work with them easily as operators. According to [1] the zero-lag fourth order cumulant of the random process x_i is defined as:

$$C_4 x_i = C(x_i, x_i, x_i, x_i) \tag{3.1}$$

and is expressed in terms of moments [1,2]:

$$C_4 x_i = E(x_i^4) - 3E^2(x_i^2) \tag{3.2}$$

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Consequently, the three zero-lag fourth-order output cross-cumulants of model (2) are defined as:

$$\begin{aligned}\Gamma_{31} &= C(s_1,s_1,s_1,s_2)\\ \Gamma_{13} &= C(s_1,s_2,s_2,s_2)\\ \Gamma_{22} &= C(s_1,s_1,s_2,s_2)\end{aligned} \tag{4.1}$$

and in terms of moments as:

$$\begin{aligned}\Gamma_{31} &= E(s_1^3 s_2) - 3E(s_1^2)E(s_1 s_2)\\ \Gamma_{13} &= E(s_1 s_2^3) - 3E(s_1 s_2)E(s_2^2)\\ \Gamma_{22} &= E(s_1^2 s_2^2) - E(s_1^2)E(s_2^2) - 2E^2(s_1 s_2)\end{aligned} \tag{4.2}$$

Applying linearity and multiplying by constant properties of the cumulants as operators, [1], on the system of equations (2) the following is obtained:

$$\Gamma_{31} = w_{11}^3 w_{21}\gamma_{40} + w_{12}^3 w_{22}\gamma_{04} \tag{5.1}$$
$$\Gamma_{13} = w_{11} w_{21}^3\gamma_{40} + w_{12} w_{22}^3\gamma_{04} \tag{5.2}$$
$$\Gamma_{22} = w_{11}^2 w_{21}^2\gamma_{40} + w_{12}^2 w_{22}^2\gamma_{04} \tag{5.3}$$

where γ_{40} and γ_{04} are the zero-lag input sample cumulants defined as:

$$\gamma_{40} = C(y_1,y_1,y_1,y_1)$$
$$\gamma_{04} = C(y_2,y_2,y_2,y_2)$$

The same definition of the fourth-order cumulants was also used in [4], while in [3,5] the fourth-order output cross-cumulants were used according to:

$$\Gamma_{ij} = cum_{i+j}(s_1^i, s_2^j) \qquad i,j \in \{1,2,3\} \tag{6}$$

Applying such defined cross-cumulants on the separation model based on the orthogonal transformation matrix defined as, [3,5],:

$$W = c\begin{bmatrix}1 & \theta\\ -\theta & 1\end{bmatrix} \qquad c = \frac{1}{\sqrt{1+\theta^2}}$$

the set of three polynomial equations is obtained. By using certain properties of input cumulants the solution is:

$$\begin{aligned}\theta_o &= \frac{\rho}{2} - sign(\rho)\sqrt{\rho^2/4+1}\\ \rho &= \frac{\gamma_{13}-\gamma_{31}}{\gamma_{22}}\end{aligned} \tag{8}$$

It is evident that three fourth-order input sample cross-cumulants have to be estimated to get solution θ_0.
To find w_{ij} from (5) all three equations must be equated with zero. Expressing w_{21} from (5.2) and inserting it in (5.1) and (5.3) the following is obtained:

$$w_{22}\gamma_{04}\left(w_{12}^3 - w_{11}^{8/3} w_{12}^{1/3}\left(\frac{\gamma_{40}}{\gamma_{04}}\right)^{2/3}\right) = 0 \tag{9.1}$$

$$w_{22}^2\gamma_{04}\left(w_{12}^2 + w_{12}^{2/3} w_{11}^{4/3}\left(\frac{\gamma_{40}}{\gamma_{04}}\right)^{1/3}\right) = 0 \tag{9.2}$$

It follows from (9.1):

$$w_{12} = w_{11}\sqrt[4]{\frac{\gamma_{40}}{\gamma_{04}}}. \tag{10.1}$$

and from (9.2):

$$w_{12} = w_{11}\sqrt[4]{-\frac{\gamma_{40}}{\gamma_{04}}} \tag{10.2}$$

w_{11} and w_{22} are chosen arbitrarily but from (2) it is clear that they are not allowed to be zero. Without loss of generality it can be adopted: w_{11}=1.0, w_{22}=1.0. Then for n=2 signals the following solutions are obtained :

- if the source signals have the same sign of kurtosis:

$$w_{12} = \sqrt[4]{\frac{\gamma_{40}}{\gamma_{04}}}, \quad w_{21} = -\sqrt[4]{\frac{\gamma_{04}}{\gamma_{40}}} \tag{11.1}$$

- if the source signals have different sign of kurtosis:

$$w_{12} = \sqrt[4]{-\frac{\gamma_{40}}{\gamma_{04}}}, \quad w_{21} = -\sqrt[4]{-\frac{\gamma_{04}}{\gamma_{40}}} \tag{11.2}$$

Compared with (8) it is evident that only two input sample cumulants are required to be estimated. This can be important when the signal separation problem is implemented in real time and the source signals are nonstationary. Compared to [9], the separation matrix is obtained as a solution. In [9] the direct solution for the coefficients of the mixing matrix is given for the case of non-Gaussian sources. The mixing matrix is obtained by rooting fourth-degree polynomial equation. Four fourth-order input cumulants: C_{40}, C_{04}, C_{13} and C_{22} are required to be estimated in order to obtain the mixing matrix. From the real time implementation point of view this solution appears to be very impractical. The input cumulants can be effectively computed by using recursive relations:

$$\begin{aligned}\gamma_{40}(k+1) &= M_{40}(k+1) - 3M_{20}^2(k+1)\\ \gamma_{04}(k+1) &= M_{04}(k+1) - 3M_{02}^2(k+1)\end{aligned} \tag{13}$$

where:

$$M_{40}(k+1) = \frac{1}{N}\left[y_1^4(k+1) - y_1^4(k-N+1)\right] + M_{40}(k)$$

$$M_{20}(k+1) = \frac{1}{N}\left[y_1^2(k+1) - y_1^2(k-N+1)\right] + M_{20}(k)$$

where N is the length of data record. M_{04} and M_{02} are defined analogously. By using recursive relations (13) the input sample cumulants can be estimated accurately enough with long data records without affecting the computational complexity. Actually, the computational complexity is invariant of the data record length N.

III. Experimental Results

The analytical solutions (11) have been testing on the separation of two linearly mixed frequency modulated (FM) signals produced by an electrooptical system that uses FM type of modulation to encode the space position of the light source. The first signal has 4kHz deviation, and the second one 1kHz. The photodiode signals were digitally recorded with the sampling frequency of 100 ksamples and then mixed with matrix A according to (1):

$$A = \begin{bmatrix} 1.0 & 9.9e-6 \\ 1.0 & 9.8e-6 \end{bmatrix} \tag{14}$$

with detA = -1e-7. On that way signal x_2 has been made extremely weak. To ensure the source signals to have zero mean they had to be digitally bandpass filtered in order to suppress as the DC so the spurious components. The spectrum of two signals on the second sensor is shown in figure 1. Figure 2. shows the result of the separation process on the second sensor by using neural network separator proposed in [7]. It was not able to recover the source signal x_2. Figures 3. and 4. show the separation results by using cumulant based separation algorithms according to the proposed solution (11) and to the solution given in [3,5], respectively. The length of the data record the input sample cumulants were estimated on was N=730. The proposed algorithm gives the same separation quality as the algorithm given in [3,5]. The three separators were coded in the assembly language of the digital signal processor TMS320C40, [10]. It is the latest generation of the very powerful and advanced DSP. Practically all instructions are single cycled. When implementing the neural network separator the problem arises in determining the amount of the convergence control factors. The large values of them ensure fast but very oscillatory convergence, while the small values ensure slow but smooth convergence. That becomes especially serious problem when the source signals x_1 and x_2 are nonstationary. In the context of this paper they were assumed to be stationary and convergence control factors were changed in four discrete steps. The results are given in table 1. The maximal sampling frequency is computed as the inverse of the total time for each algorithm, assuming that single cycle time is 40 ns.

TABLE 1.

Algorithm	No. of cycles	Fs [kHz]
Neural network	126	198
Cumulant after [3,5].	126	198
Cumulant after (11)	89	280

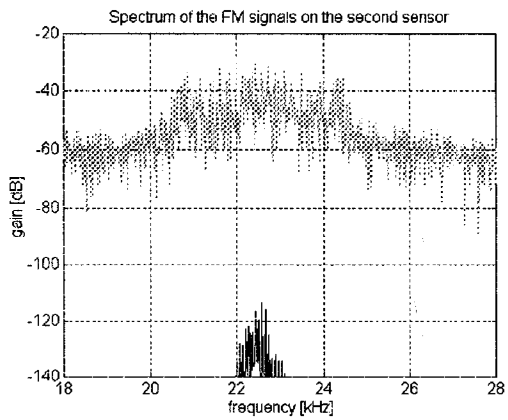

Figure 1.

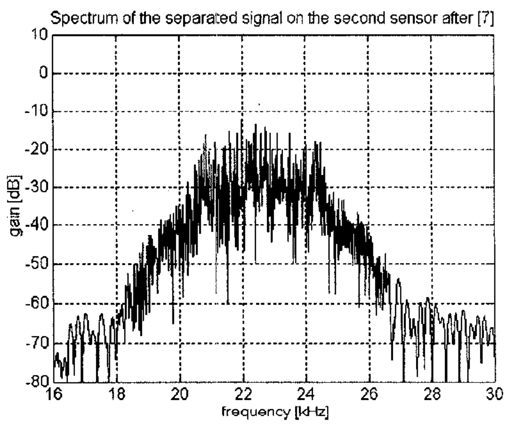

Figure 2.

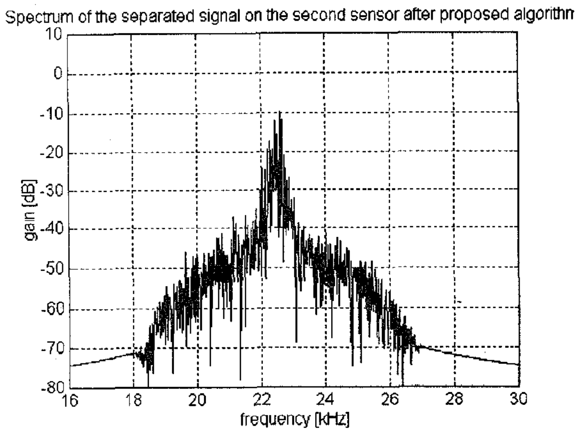

Figure 3.

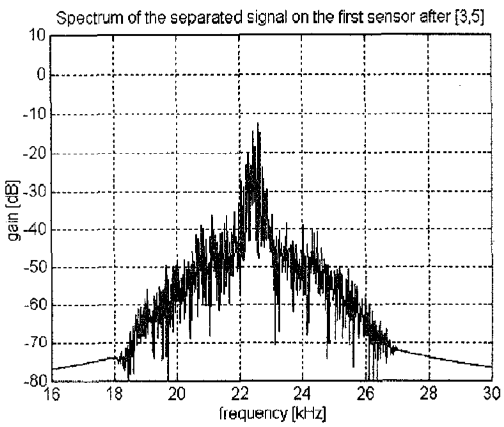

Figure 4.

IV. Conclusion And Future Work

The paper presents the alternative analytical solution of the two signals separation problem based on canceling all three fourth-order output cross-cumulants. In comparison with solution given in [3,5] the proposed solution requires that only two, instead of three, input sample cumulants have to be estimated. The proposed algorithm has the same quality of separation as algorithm proposed in [3,5], having at the same time 30% less computational complexity. When the source separation problem is implemented in real time this feature can be significant. It has been also shown that for extremely badly scaled signals neural network separators, contrary to the cumulant based separators, fail to separate the source signals. The future work will be directed toward experimental research related to the construction of the electrooptical system for space localization of two light sources by using blind signal separation theory.

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