Blind Signal Deconvolution as an Instantaneous Blind Separation of Statistically Dependent Sources

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Abstract. We propose a novel approach to blind signal deconvolution. It is based on the approximation of the source signal by Taylor series expansion and use of a filter bank-like transform to obtain multichannel representation of the observed signal. Currently, as an *ad hoc* choice a wavelet packets filter bank has been used for that purpose. This leads to multi-channel instantaneous linear mixture model (LMM) of the observed signal and its temporal derivatives converting single channel blind deconvolution (BD) problem into instantaneous blind source separation (BSS) problem with statistically dependent sources. The source signal is recovered provided it is a non-Gaussian, non-stationary and non- independent identically distributed (i.i.d.) process. The important property of the proposed approach is that order of the channel filter does not have to be known or estimated. We demonstrate viability of the proposed concept by blind deconvolution of the speech and music signals passed through a linear low-pass channel.

Keywords: Blind deconvolution, Blind source separation, Independent component analysis, Instantaneous mixture model, Statistically dependent sources.

1 Introduction

The problem of single channel BD is to reconstruct the original signal from its filtered version also termed observed signal, where only observed signal is available. Neglecting the noise term the process is modeled as a convolution of the unknown causal channel impulse response $h(t)$ with an original source signal $s(t)$ as:

$$x(t) = \sum_{\tau=0}^{T} h(\tau)s(t-\tau) \tag{1}$$

where T denotes the order of the channel filter. Standard algorithms for blind deconvolution are capable of recovering source signal $s(t)$ based on the observed signal $x(t)$ only, provided that $s(t)$ is a non-Gaussian i.i.d. process, [1]. We shall demonstrate here that proposed concept is capable of blind deconvolution of signals with colored statistics such as speech. The original signal $s(t\text{-}\tau)$ can be approximated by Taylor series expansion around $s(t)$ giving:

$$s(t-\tau) = \sum_{n=0}^{N} \left((-\tau)^n / n! \right) s^{(n)}(t) + \text{H.O.T.} \tag{2}$$

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where $s^{(n)}(t)$ denotes *n-th* order temporal derivative of *s*(*t*) and H.O.T. denotes higher-order-terms. It is assumed that $s^{(0)}(t) = s(t)$. Inserting (2) into (1) yields:

$$x(t) \cong \sum_{n=0}^{N} a_{1(n+1)} s^{(n)}(t) \tag{3}$$

where $a_{11} = \sum_{\tau=0}^{T} h(\tau)$, $a_{12} = -\sum_{\tau=0}^{T} \tau h(\tau)$, $a_{13} = \sum_{\tau=0}^{T} \left(\tau^2 / 2\right) h(\tau)$, etc. Evidently, quality of the approximations (2) and (3) depends on the number of terms in the Taylor series expansion of the source signal *s*(*t*). However, *x*(*t*) in (3) can be also obtained as an inverse Fourier transform of the expression $H(j\omega)S(j\omega)$ where $H(j\omega)$ and $S(j\omega)$ respectively represent Fourier transforms of the channel impulse response and source signal. Owing to the fact that *h*(*t*) is an aperiodic sequence $H(j\omega)$ is obtained as

$$H(j\omega) = \sum_{\tau=0}^{T} h(\tau) e^{-j\omega\tau} \cong \sum_{\tau=0}^{T} h(\tau) - j\omega \sum_{\tau=0}^{T} \tau h(\tau) + \frac{(j\omega)^2}{2} \sum_{\tau=0}^{T} \tau^2 h(\tau) \tag{4}$$

that yields

$$X(j\omega) \cong \left(\sum_{\tau=0}^{T} h(\tau) \right) S(j\omega) - \left(\sum_{\tau=0}^{T} \tau h(\tau) \right) j\omega S(j\omega) + \left(\sum_{\tau=0}^{T} \tau^2 h(\tau) \right) \frac{(j\omega)^2}{2} S(j\omega) \tag{5}$$

Evidently, number of terms in the expansions (4) and (5) depends on the property of the channel: size of the support *T* of the impulse response *h*(*t*), but also on the property of the signal: size of its support Ω in the frequency domain i.e. $|S(j\omega)| \cong 0$ for $\omega > \Omega$. For example, for either *T*=0 or Ω =0 relation (3) and inverse Fourier transform of (5) yield the same result. Thus, channels with the maximal delay that is small relative to the coherence time of the signal, i.e. *T*<<(2π/ Ω), will demand small number of terms, *N*, in the approximation (3) and vice versa.

Taylor series expansion has already been used in [2]-[7] to convert multichannel convolutive BSS problem into instantaneous BSS problem. Two cases can be distinguished. In [2]-[5] authors assumed sensor array that is smallest than the shortest wavelength of the sources. This allows to keep only the first order derivative in the Taylor series expansion in Eq.(2). This is due to the fact that delay is defined relative to the center of the array and is therefore always smaller than the coherence time of the sources. Under this assumption another array that calculates spatial gradients of the observed signal converts the convolutive BSS problem into instantaneous BSS problem with the first order temporal derivatives of the source signals acting as sources. Once they are recovered by instantaneous ICA, the true sources are obtained by their temporal integration. In [6] and [7] Taylor series expansion is also used to convert multichannel convolutive BSS problem into instantaneous BSS problem. In [6] it is assumed that delay *T* is smaller than the coherence time of the source signals which allows to use only first order temporal derivative in the Taylor series expansion Eq.(2). Assuming that signal and its first order derivative are statistically independent, that is actually proven for stationary signals only [8][9], the instantaneous BSS problem is solved by some of the standard ICA methods. However, assumption that delay is smaller than the coherence time of the source signals is too restrictive for realistic reverberant environments. That was realized in [7]. In that case higher order temporal derivatives exist in the Taylor series expansion Eq.(2), and they are statistically dependent. An algorithm is derived in [7] for grouping dependent sources and extracting source signals from each group.

The algorithm proposed here solves single channel BD problem by converting it into instantaneous BSS problem with statistically dependent sources. No special assumption is made on the amount of delay. Thus, higher order derivatives in the Taylor series expansion are allowed. The problem of their statistical dependence is solved by means of independence enhancement technique, which is based on innovations of the multichannel version of the observed signal. However, another transform such as high-pass filtering, [10], may be used for independence enhancement purpose as well.

A BD capable of recovering temporally dependent signals is derived in [11]. It is based on the measure of temporal predictability and argumentation that an output of the low-pass channel is smoother and therefore more predictable than the input to the channel. Thus, the BD problem is formulated as temporal predictability minimization problem and numerically solved as general eigenvalue problem. Equivalent solution of the instantaneous BSS problem by looking for maximum of the temporal predictability is defined in [12]. In relation to the proposed Taylor series expansion BD method, the temporal predictability approach suffers from the fact that order of the deconvolution filter has to be defined based on some *a priori* knowledge. Because the order of the generalized eigenvalue problem equals the order of the deconvolution filter the temporal predictability based algorithm can become numerically very demanding. Temporal predictability itself is defined for deconvolved signal $y(t)=\sum_{k=0}^{M} w(k)x(t-k)$ as:

$$F\left(\{y(t)\}\right)=\log\frac{V\left(\{y(t)\}\right)}{U\left(\{y(t)\}\right)}=\log\frac{\sum_{t}^{t_{\max}}\left(\bar{y}(t)-y(t)\right)^2}{\sum_{t}^{t_{\max}}\left(\tilde{y}(t)-y(t)\right)^2} \tag{6}$$

where V reflects the extent to which y(t) is predicted by long term moving average $\bar{y}(t)$ and U reflects the extent to which y(t) is predicted by short term moving average $\tilde{y}(t)$, [12][11].

2 Formulation of the Instantaneous Linear Mixture Model

We now apply a filter bank-like transform on (3) in order to obtain a multichannel representation, **x**, of the observed signal $x(t)$. It is the matter of further analysis to find out which type of the transform is optimal. Here, in order to illustrate the concept, as an *ad hoc* choice we have used a non-decimated wavelet packets filter bank with two decomposition levels that results in L=6 filters. In order to have clear notation let us introduce $x_1(t)=x(t)$. When filters are applied on observed signal $x(t)$ we obtain:

$$x_{l+1}(t)\cong a_{l+1,1}s(t)+a_{l+1,2}s^{(1)}(t)+a_{l+1,3}s^{(2)}(t) \quad l=1,..,L \tag{7}$$

where $a_{l+1,1}=\sum_{\tau=0}^{\bar{T}}\bar{h}_l(\tau)$, $a_{l+1,2}=-\sum_{\tau=0}^{\bar{T}}\tau\bar{h}_l(\tau)$, $a_{l3}=\sum_{\tau=0}^{\bar{T}}\left(\tau^2/2\right)\bar{h}_l(\tau)$, where $\bar{h}_l(t)$ represents convolution of the appropriate l-th filter with $h(t)$, $\bar{T}=T+M+2$ and M is an order of the filter. Observed signal and its filtered versions can be represented in a form of the following instantaneous LMM:

$$\mathbf{x}(t)=\begin{bmatrix} x_1(t) \\ x_2(t) \\ \cdots \\ x_{L+1}(t) \end{bmatrix} \cong \begin{bmatrix} a_{11} & a_{12} & a_{13}\cdots\cdots a_{1,N+1} \\ a_{21} & a_{22} & a_{23}\cdots\cdots a_{2,N+1} \\ \cdots & \cdots & \cdots \\ a_{L+1,1} & a_{L+1,2} & a_{L+1,3}\cdots\cdots a_{L+1,N+1} \end{bmatrix} \begin{bmatrix} s(t) \\ s^{(1)}(t) \\ s^{(2)}(t) \\ \cdots\cdots \\ s^{(N)}(t) \end{bmatrix} = \mathbf{As}(t) \tag{8}$$

where $\mathbf{x}\in\mathbf{R}^{(L+1)\times K}$, $\mathbf{A}\in\mathbf{R}^{(L+1)\times(N+1)}$, $\mathbf{s}\in\mathbf{R}^{(N+1)\times K}$, K represents number of samples and N represents an unknown number of temporal derivatives of the source signal. We have used inspection of the singular values of the sample data covariance matrix $\hat{\mathbf{R}}_{\mathbf{xx}}=(1/K)\mathbf{xx}^{\mathrm{T}}$ to estimate overall number of sources, N+1. ICA algorithms can be applied to the LMM given by Eq. (8) in order to extract the source signal $s(t)$, with the benefits that the order T of the channel impulse response $h(t)$ is absorbed in the mixing matrix **A** and does not have to be known or estimated. The *source* signals have to be non-Gaussain and statistically independent but not i.i.d. This has important practical consequence because BD of signals with colored statistics is possible. This is demonstrated in the section 4 where simulation results are presented.

3 Statistical Properties of the Source Signal: Implications to Deconvolution Results

We reproduce here results and conditions from [8][14] necessary for the stochastic differentiability of the random source signal $s(t)$. We emphasize that conclusions drawn from this analysis can in principle be generalized to blind image deconvolution problem due to the existence of the space filling curves (Peano-Hillbert curves) that enable 2D to 1D mapping and vice versa by preserving local or neighborhood statistics [13]. First we present two important results that relate (non-)stationarity and linear signal representation. If the signal $s(t)$ is stationary it can be represented by the linear time invariant generative model:

$$s(t)=\sum_{\nu=0}^{\infty} b(\nu)\varepsilon(t-\nu) \tag{9}$$

where $\varepsilon(t)$ is an i.i.d. driving signal. If the signal $s(t)$ is non-stationary the linear signal model becomes time variant:

$$s(t)=\sum_{\nu=0}^{\infty} b(t,\nu)\varepsilon(t-\nu) \tag{10}$$

First order derivative $s^{(1)}(t)$ of the stationary signal $s(t)$ is defined if the first order derivative of the autocorrelation function at the time lag zero is zero i.e. $\rho_s^{(1)}(0)=0$, [8]. $\rho_s^{(1)}(0)$ is always zero for non-i.i.d. process due to symmetry of $\rho_s(\tau)$. According to [8] the stronger condition for the existence of $s^{(1)}(t)$ is $\rho_s^{(2)}(0)\neq 0$. If this is true then from [9] it is also true $\rho_s^{(2)}(\tau)\neq 0, \forall\tau$. Analogously, condition for existence of $s^{(2)}(t)$

assumes $\rho_s^{(3)}(0) \neq 0$. If the first order derivative of the stationary signal $s(t)$ exists then [8]:

$$E\left[s(t)s^{(1)}(t)\right] = 0 \tag{11}$$

where E represents mathematical expectation. We now interpret these results for the three types of the source signal $s(t)$.

Source signal is a stationary i.i.d. process. In this case a condition $\rho_s^{(1)}(0) = 0$ is not fulfilled. The reason is that autocorrelation function of the i.i.d. process is delta function i.e. $\rho_s(\tau) = \sigma_s^2 \delta_\tau$. Therefore, Taylor series expansion (2) for such a signal does not exist. Consequently, the LMM model (8) also does not exist. Thus, i.i.d. signals can not be blindly deconvolved by the proposed algorithm. However, this is not a drawback since a number of blind deconvolution methods solve this problem, [1][15].

Source signal is a stationary non-i.i.d. process. As it has been said such signal has first order derivative. Under previously defined conditions second order derivative also exists. However, we have to emphasize that stationary signals, that are represented by linear time invariant generative signal model (9), can also not be blindly deconvoloved by the proposed algorithm. Assuming that $b(t)$ represents impulse response of the linear time invariant signal generative model, it is impossible to distinguish the channel filter $h(t)$ from the linear convolution of the channel filter and modeling filter $h(t)*b(t)$. Thus, proposed algorithm will deconvolve the i.i.d. driving sequence $\varepsilon(t)$, i.e. the algorithm will have the whitening effect on the stationary non-i.i.d. signal.

Source signal is a non-stationary and non-i.i.d. process. Although, conditions required for stochastic differentiability are derived for stationary signals only we can use the linear generative model of the non-stationary signal (10) and derive derivatives of the non-stationary signal $s(t)$ provided that time varying filter $b(t, \nu)$ is stationary with respect to the independent variable t. In such a case we define:

$$s^{(m)}(t) = \sum_{\nu=0}^{\infty} b^{(m)}(t,\nu)\varepsilon(t-\nu) \tag{12}$$

where $b^{(m)}(t,\nu) = \left(d^m b(t,\nu)/dt^m\right)$. Thus, Taylor series expansion (2) and the LMM (8) do exist. However, we can not make conclusion regarding statistical independence between $s(t)$, $s^{(1)}(t)$, $s^{(2)}(t)$, etc, as it was the case with a stationary signal, (11). Thus, it is justified to use some of the methods derived to enhance statistical independence between the hidden variables in the LMM (8). One of them that is computationally efficient is based on innovations, [16]. It is known that innovations are more non-Gaussian and more statistically independent than original processes. These conditions are of essential importance for the success of the ICA algorithms. Innovation process of the hidden components of **s** is

$$\tilde{s}_n(t) = s_n(t) - E\left[s_n(t) \middle| t, s_n(t-1), s_n(t-2), \ldots\right] \quad s_n \in \left\{s, s^{(1)}, s^{(2)}, \ldots\right\} \tag{13}$$

where the second term in Eq.(13) represents conditional expectation. If both sides of (13) are multiplied by the unknown basis matrix **A** we obtain

$$\tilde{\mathbf{x}}(t) = \mathbf{A}\tilde{\mathbf{s}}(t) \tag{14}$$

Eq.(14) implies that innovations preserve the basis matrix **A**. The innovations based multichannel model (14) enables more accurate estimation of the mixing matrix **A** by means of ICA algorithms, than when ICA algorithms are applied directly on the LMM (8). The expectation is in practice replaced by the autoregressive (AR) model of the finite order yielding:

$$\tilde{x}_l(t) = \sum\nolimits_{j=0}^{J} g_j x_l(t-j) \tag{15}$$

where J represents order of the AR model and g_0=1. The coefficients of the prediction-error filter g_j are efficiently estimated by means of Levinson's algorithm, [17]. We identify for the LMM model (8) L+1 filters and obtain the prediction-error filter in (15) as an average of all identified filters. Hidden variables are then recovered by applying the Moor-Penrose pseudoinverse $\mathbf{A}^{\dagger}$ on the originally observed process **x**. The temporal predictability measure Eq.(6) could be used as a criteria for the selection of the recovered source signal $\hat{s}(t)$ after solution of BSS problem (8).

4 Simulation Results

We have conducted the following experiments: BD of the female speech signal and BD of the choir singing passed through a lowpass channels. 2nd order Butterworth lowpass filter has been used to model the channel response. Figure 1 shows one hundred time points of the true female speech source signal, signal recovered by temporal predictability based algorithm, [12] and signal recovered by the proposed algorithm. For temporal predictability based algorithm we have shown the best result obtained after experimenting with several values for the order of the deconvolution filter. Normalized correlation coefficients between the source and mixed signal, source signal and signal recovered by the proposed algorithm, and source signal and

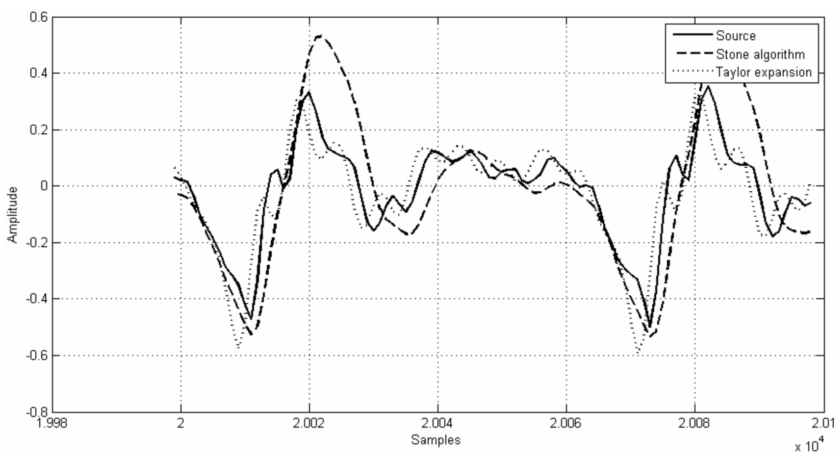

Fig. 1. One hundred time samples of the source signal (solid), signal recovered by temporal predictability based algorithm (dashed), [12], and proposed algorithm based on the Taylor series expansion (dotted)

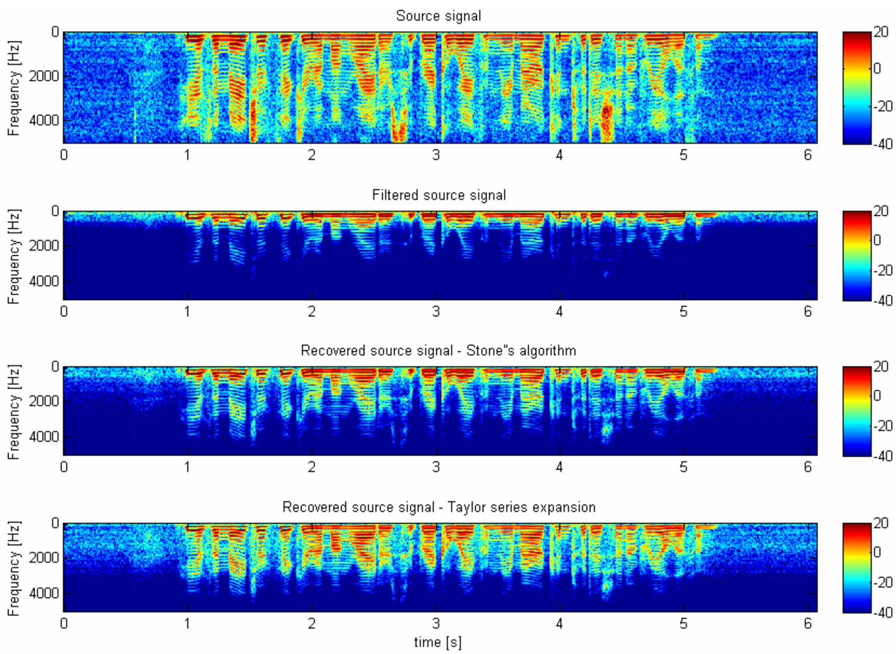

Fig. 2. From top to bottom: spectrograms of the source signal, observed signal, signal recovered by temporal predictability based algorithm, [12], and signal recovered by the proposed algorithm

signal recovered by algorithm [12] are respectively: 0.71774, 0.88658 and 0.75476. Spectrograms of these signals are shown in Figure 2. Regarding the choir-singing signal the normalized correlation coefficients in the same order as before were 0.5276, 0.86152 and 0.84015.

5 Conclusion

Novel single channel BD algorithm has been formulated. It is based on the approximation of the source signal by Taylor series expansion and use of a filter bank-like transform to yield a multichannel representation of the observed single-sensor signal. This yields instantaneous LMM and converts the single channel BD problem into instantaneous BSS problem with statistically dependent sources with the important property that channel order does not have to be known. It has been shown that signal amenable for BD by proposed method must be non-stationary and non-i.i.d. non-Gaussian process. As yet unresolved issues remain: optimality of the linear transforms used to yield a multivariate representation of the observed signal and efficiency of the linear transforms used to enhance statistical independence among hidden variables of the LMM. The later issue might affect performance of the proposed algorithm when degradations are strong as it can be expected for real acoustic channels.

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