Performance Evaluation of the Second Order and Fourth Order Statistics Based Root MUSIC Algorithms in the Presence of Mutual Coupling

Ivica Kopriva*, Wasyl Wasylkiwskyj and Hossam Abdallah
Department of Electrical and Computer Engineering,
The George Washington University
801 22nd St. NW Room 615, Washington DC 20052
e-mail: (ikopriva, wasylkiw, hossam)@gwu.edu

Abstract

Direction finding (DF) algorithms based on polynomial rooting implicitly assume equality among the element radiation patterns. However, in real antenna arrays this equality is violated due to the phenomenon known as mutual coupling what can significantly deteriorate accuracy of the root DF algorithms. In this paper we compare robustness of the second order (SO) and fourth order (FO) statistics based Root MUSIC algorithms in the presence of the various levels of mutual coupling among the elements of the uniform linear array (ULA). It has been demonstrated that FO Root MUSIC algorithm exhibits significantly higher level of robustness in the presence of mutual coupling when number of sources is greater than one.

Introduction

Polynomial rooting based super-resolution DF algorithms such as Root-MUSIC [2] offer computational efficiency in relation to the search based DF methods [1] but assume equal element radiation patterns in the terminated array environment. This is true for SO statistics based Root-MUSIC method [2] as well as for FO statistics based formulation of the Root-MUSIC method [3]. Because of mutual coupling the element patterns equality condition is generally not met so that root-based DF algorithms necessarily suffer a loss in accuracy unless some form of compensation is employed [4]-[6]. Mutual coupling compensation techniques are briefly discussed in second section. SO and FO statistics based Root-MUSIC algorithms are introduced in third section. Fourth section compares these two algorithms when 1, 2 and 3 signals were impinging on the 4-element ULA for the scenarios: (i) without coupling; (ii) with 4 dipole antennas; (iii) with 4 active and 3 passive dipole elements on each side of the array. Conclusion is given in section five.

Linear antenna array model

The problem of estimating the Direction Of Arrival (DOA) of L independent plane waves incident on a linear equispaced array of N sensors is described with:

$$\mathbf{z}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{v}(t) \qquad (1)$$

where $\mathbf{z}(t)$ is a complex column vector comprised of N signals at the output of ULA; $\mathbf{A}$ is NxL steering matrix of the linear array comprised of the L column vectors $\mathbf{a}(\Omega_l)$ corresponding with the DOA of the *l-th* source signal; $\mathbf{s}(t)$ is a column vector comprised of the L source signals incident on the linear array and $\mathbf{v}(t)$ represents additive Gaussian noise. It is assumed that additive noise is not mutually correlated. The steering vector of the ULA required by polynomial rooting DF algorithms is given with

$$\mathbf{a}(\Omega_l) = \hat{f}(\Omega_l)\begin{bmatrix}1 & e^{jk_0 d\Omega_l} & e^{jk_0 2d\Omega_l} \ldots & e^{jk_0 (N-1)d\Omega_l}\end{bmatrix}^{\mathrm{T}} \qquad (2)$$

where $\Omega_l = \sin(\theta_l)\sin(\varphi_l)$, θ_l is elevation and φ_l is azimuth, $k_0 = 2\pi/\lambda$ is a free space wave number evaluated at the receiver local oscillator frequency, λ is a wavelength, d is an inter-element spacing expressed as a fraction of λ. It is assumed that elements are placed along the y-axis with the first element placed at the origin. In (2) $\hat{f}(\Omega_l)$ is a constant multiplier that is obtained when equality among the element radiation patterns is assumed i.e. $\hat{f}_n(\Omega_l) = \hat{f}(\Omega_l),\ \forall n = 1,2,..,N$. However, due to the mutual coupling the element radiation patterns in the terminated array environment will generally deviate from equality. Some techniques for mutual coupling compensation are discussed in [4]-[6]. In [4] a decoupling transform is designed and applied on the digital baseband signals obtained at the output of the 4-element array. In order to implement decoupling transform a scattering matrix of the whole receiving system (receiving array and donwconverter) must be measured. In [5] a method proposed to compensate for effects of mutual coupling is equivalent to the method discussed in [4]. The difference is that in order to identify coupling matrix element patterns must be measured. In [6] an "infinite" array concept is proposed exploiting symmetry of the ULA. Thus, by adding a few passively terminated elements at both ends of a finite length array some degree of approximation of the infinite array environment can be achieved resulting in greater equality of the element radiation patterns. In the numerical example reported here we have used the passive element approach to element pattern equalization [6].

SO and FO statistics based Root MUSIC Algorithm

It is assumed here that source signals are non-Gaussian and statistically independent in the sense of FO statistics [3][7][8]. In formulation of the MUSIC algorithm [1] the nonnegative function

$$\Lambda(\Omega) = \mathbf{a}(\Omega)^H \mathbf{E}_v \mathbf{E}_v^H \mathbf{a}(\Omega) \qquad (3)$$

called pseudo-spectrum is used to find DOA. In (3) $\mathbf{E}_v$ represents matrix of eigenvectors that span the noise subspace and are obtained from the eigenvalue decomposition of the sample data covariance matrix $\hat{\mathbf{R}}_{zz} = (1/T)\sum_{t=1}^{T}\mathbf{z}(t)\mathbf{z}(t)^{\mathrm{H}}$. For decoupled linear array (2) $\Lambda(\Omega)$ can be written in a polynomial form [2] as

$$\Lambda(z) = z^{-(N-1)} P_{2N-2}(z) \qquad (4)$$

where $z = e^{i\Omega}$ and $P_{2N-2}(z)$ is the *2N-2* degree polynomial in *z*. From (4) DOA are found from the *L* complex roots of the polynomial $P_{2N-2}(z)$ that are closest to the unit circle as

$$\Omega_l = angle(z_l)/k_0/d \qquad l = 1,2,\ldots,L \qquad (5)$$

By analogy with the SO MUSIC pseduospectrum (3), the FO version is formulated as [8]

$$\Lambda(\Omega) = \left(\mathbf{a}(\Omega)\otimes\mathbf{a}^*(\Omega)\right)^H \mathbf{E}_v \mathbf{E}_v^H \left(\mathbf{a}(\Omega)\otimes\mathbf{a}^*(\Omega)\right) \qquad (6)$$

where in (6) ⊗ denotes tensorial or Kronecker's product, '*' denotes complex conjugate operation and $\mathbf{E}_v$ represents matrix of eigenvectors that correspond with noise subspace and are obtained from the eigenvalue decomposition of the $N^2 x N^2$ sample data quadricovariance matrix $\mathbf{Q}_{zz}$ with the entries

$$\mathbf{Q}_{zz}(r,q) = cum\left(z_i(t), z_k^*(t), z_l^*(t), z_m(t)\right) = \gamma_{4,s} \hat{f}_i \hat{f}_k^* \hat{f}_l^* \hat{f}_m \exp(j(o-p)k_0 d\Omega) \quad (7)$$

where in (7) $o=i+m$, $p=k+l$ and $\gamma_{4,s}$ represents FO cumulant of the source signal $s(t)$. The coordinates (r,q) in (7) are obtained from the $C^4 \rightarrow C^2$ mapping according to scheme $r=N(i-1)+k$, $q=N(l-1)+m$ where C^D denotes field of complex numbers of the dimension D. This mapping is necessary because quadricovariance is originally a four dimensional tensor, [8]. Assuming equality among the element radiation patterns the FO pseudospectrum (6) can be written as

$$\Lambda(z) = z^{-2(N-1)} P_{4N-4}(z) \quad (8)$$

with $z = e^{j\Omega}$. In accordance with (5) DOA are found from L pairs of complex roots of the polynomial $P_{4N-4}(z)$ that are closest to the unit circle. A FO statistics approach to DF offers two principal advantages over SO statistics based DF: (a) the FO methods are blind w.r.t. additive Gaussian noise due to the fact that FO cumulants of the Gaussian process are asymptotically equal to zero [7]; (b) the FO methods increase effective aperture of the antenna array in relation to the SO methods [3][7][8]. In comparison between (4) and (8) it can be seen that maximal number of DOA that can be estimated by the SO Root MUSIC is N-1 and with the FO Root MUSIC 2N-2.

Numerical results

Performance of the SO and FO Root MUSIC algorithms is compared in the presence of the various levels of mutual coupling assuming 4-element ULA of half wavelength dipole antennas for the cases when 1 source from direction of 50^0, 2 sources from directions of 50^0 and 60^0 and 3 sources from directions of 50^0, 60^0 and 70^0 were impinging on the array. The QPSK modulation was assumed. Sample size was 10000 samples and SNR per source was 25dB. On that way error in DOA estimation due to the low SNR was eliminated because FO method would be in favorable position due to the fact that it does not see additive Gaussian noise. Three scenarios were tested: (i) ULA without coupling; (ii) ULA with 4 coupled dipole antennas; (iii) ULA with 4 active and 3 passive dipole elements on each side of the array. Element radiation patterns were calculated numerically by using the WIPL code [9]. Ten runs were executed to obtain the mean values of the DOA estimate for each of described cases. A MSE $\varepsilon = \sqrt{\sum_{l=1}^{L}\left(\hat{\Omega}_l - \Omega_l^*\right)^2 / L}$ was used as a performance measure where $\hat{\Omega}$ denotes estimated DOA and Ω^* denotes true DOA and $L \in \{1,2,3\}$. Results are shown in Fig. 1 for the cases (ii) and (iii) respectively. Results for the SO Root MUSIC are shown in black color and for the FO Root MUSIC in gray color. In a case of ideal array without coupling, case (i), MSE was less than 0.1^0 for both SO and FO Root MUSIC and for any L. As could be seen in Fig.1 the FO Root MUSIC method exhibits higher level of robustness in relation to the presence of mutual coupling among the array elements when more than one source was impinging on the array. The error is significantly reduced when array was decoupled by means of passive elements, in which case both methods exhibited similar accuracy. The greater robustness of the FO Root MUSIC is due to the increased array aperture.

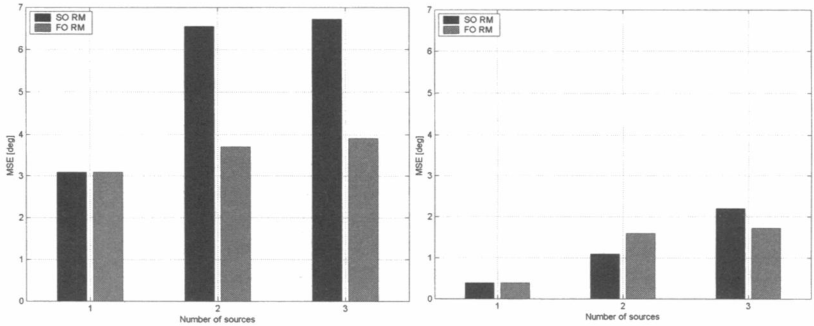

Figure 1. MSE with coupling (left) and with decoupled array using 4 active and 6 passive elements (right). Black-SO Root MUSIC; Gray-FO Root MUSIC.

Conclusion

Performance of the SO statistics and FO statistics based Root MUSIC algorithms was compared in the presence of various levels of mutual coupling among the elements of the ULA for different number of sources. In the presence of mutual coupling error in DOA estimation obtained by the FO Root MUSIC was significantly smaller than by the SO Root MUSIC when number of sources was greater than one. For decoupled array by means of passive elements attached to the end fires of the ULA both SO and FO Root MUSIC methods exhibited the similar performance.

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