# Lecture IX

# Independent component analysis for linear dynamic (convolutive) problems

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# Course outline

- Motivation with illustration of applications (lecture I)
- Mathematical preliminaries with principal component analysis (PCA)? (lecture II)
- Independent component analysis (ICA) for linear static problems: information-theoretic approaches (lecture III)
- ICA for linear static problems: algebraic approaches (lecture IV)
- ICA for linear static problems with noise (lecture V)
- Dependent component analysis (DCA) (lecture VI)

# Course outline

- Underdetermined blind source separation (BSS) and sparse component analysis (SCA) (lecture VII/VIII)
- Nonnegative matrix factorization (NMF) for determined and underdetermined BSS problems (lecture VIII/IX)
- BSS from linear convolutive (dynamic) mixtures (lecture X/XI)
- Nonlinear BSS (lecture XI/XII)
- Tensor factorization (TF): BSS of multidimensional sources and feature extraction (lecture XIII/XIV)

# Seminar problems

1. Blind separation of two uniformly distributed signals with maximum likelihood (ML) and AMUSE/SOBI independent component analysis (ICA) algorithm. Blind separation of two speech signals with ML and AMUSE/SOBI ICA algorithm. **Theory, MATLAB demonstration and comments of the results.**

2. Blind decomposition/segmentation of multispectral (RGB) image using ICA, dependent component analysis (DCA) and nonnegative matrix factorization (NMF) algorithms. **Theory, MATLAB demonstration and comments of the results.**

3. Blind separation of acoustic (speech) signals from convolutive dynamic mixture. **Theory, MATLAB demonstration and comments of the results.**

# Seminar problems

4. Blind separation of images of human faces using ICA and DCA algorithms (innovation transform and ICA, wavelet packets and ICA) **Theory, MATLAB demonstration and comments of the results.**

5. Blind decomposition of multispectral (RGB) image using sparse component analysis (SCA): clustering + $L_p$ norm ( $0<p\leq 1$) minimization. **Theory, MATLAB demonstration and comments of the results.**

6. Blind separation of four sinusoidal signals from two static mixtures (a computer generated example) using sparse component analysis (SCA): clustering + $L_p$ norm ( $0<p\leq 1$) minimization in frequency (Fourier) domain. **Theory, MATLAB demonstration and comments of the results.**

# Seminar problems

7. Blind separation of three acoustic signals from two static mixtures (a computer generated example) using sparse component analysis (SCA): clustering + $L_p$ norm ( $0<p\leq 1$) minimization in time-frequency (short-time Fourier) domain. **Theory, MATLAB demonstration and comments of the results.**

8. Blind extraction of five pure components from mass spectra of two static mixtures of chemical compounds using sparse component analysis (SCA): clustering a set of single component points + $L_p$ norm ( $0<p\leq 1$) minimization in m/z domain. **Theory, MATLAB demonstration and comments of the results.**

9. Feature extraction from protein (mass) spectra by tensor factorization of disease and control samples in joint bases. Prediction of prostate/ovarian cancer. **Theory, MATLAB demonstration and comments of the results.**

# Blind source separation

A theory for blind signal recovery from multichannel observation requiring minimum of *a priori* information.

**Problem:**

$$\mathbf{X}=\mathbf{AS} \quad \mathbf{X}\in R^{NxT},\ \mathbf{A}\in R^{NxM},\ \mathbf{S}\in R^{MxT}$$

**Goal:** find **A** and **S** based on **X** only.

Solution $\mathbf{X}=\mathbf{AT}^{-1}\mathbf{TS}$ must be characterized with $\mathbf{T}=\mathbf{P}\Lambda$ where **P** is permutation and $\Lambda$ is diagonal matrix i.e.: $\mathbf{Y} \cong \mathbf{P}\Lambda\mathbf{S}$

A. Cichocki, S. Amari, "Adaptive Blind Signal and Image Processing," John Wiley, 2002.

# Independent component analysis (ICA)

- Number of mixtures *N* must be greater than or equal to *M*.

- source signals $s_i(t)$ must be statistically independent.

$$\mathrm{p}(\mathbf{s}) = \prod_{m=1}^{M} p_m(s_m)$$

- source signals $s_m(t)$, except one, must be non-Gaussian.

$$\{C_n(s_m) \neq 0\}_{m=1}^{M} \quad \forall n > 2$$

- mixing matrix **A** must be nonsingular.

$$\mathbf{W} \cong \mathbf{A}^{-1}$$

A. Hyvarinen, J. Karhunen, E. Oja, "Independent Component Analysis," John Wiley, 2001.

# ICA for convolutive mixtures

In many situations related to acoustics and data communications we are confronted with multiple signals received from a multipath mixture. Sometimes, this is known under the popular name of *cocktail- party* problem.

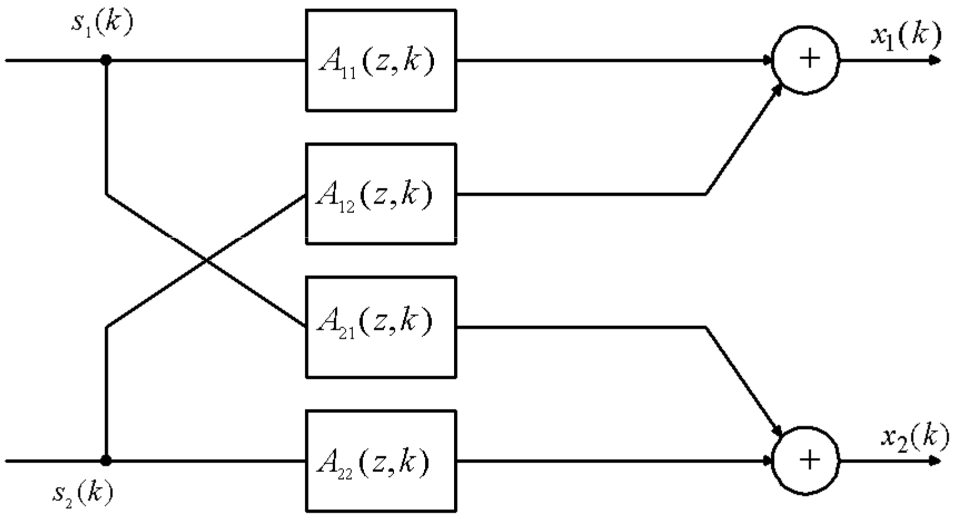
The instantaneous linear mixture model **x**=**As** is not valid anymore. Instead, the convolutive model needs to be assumed:

$$\mathbf{x}_i(n) = \sum_{j=1}^{M}\sum_{l=0}^{L} a_{ij}(l)s_j(n-l) \quad i = 1...N$$

Thus, convolutive mixture is described by a mixing matrix whose elements are the individual impulse responses (in time domain) or transfer functions (in frequency domain) between a source and a sensor.

# ICA for convolutive mixtures

When both mixing matrix and sources are unknown the problem is referred to as the multichannel blind deconvolution (MBD) problem[1-5].




A Convolutive model for 2x2 system:

$$x_i(n) = \sum_{j=1}^{2} \sum_{l=0}^{L} a_{ij}(l) s_j(n-l) \qquad i = 1, 2$$

1. A. Hyvarinen, J. Karhunen and E. Oja, *Chapter 19* in Independent Component Analysis, J. Wiley, 2001.

2. A. Cichocki, S. Amari, *Chapter 9* in Adaptive Blind Signal and Image Processing – Learning Algorithms and Applications, J. Wiley, 2002.

3. M. Castella, A. Chevreuil, J.-C. Pesquet, *Chapter 8* in Handbook of Blind Source Separation, Academic Press, P. Comon and Ch. Jutten editors, 2010.

4. R. H. Lambert and C.L. Nikias, *Chapter 9* in Unsupervised Adaptive Filtering – Volume I Blind Source Separation, S.Haykin, ed., J. Wiley, 2000.

5. S.C. Douglas and S. Haykin, *Chapter 3* in Unsupervised Adaptive Filtering – Volume II Blind Deconvolution, S. Haykin, ed., J. Wiley, 2000.

# ICA for convolutive mixtures

$$\text{Multichannel Blind Deconvolution}$$
$$=$$
$$\underbrace{\text{Blind Source Separation}}_{\text{solves interchannel interference}} + \underbrace{\text{Single Channel Blind Deconvolution}}_{\text{solves intersymbol interference}}$$

Single channel blind deconvolution problem is also referred to as blind equalization. The unknown source signal that is input to a single-input-single-output (SISO) system is required to be i.i.d. non-Gaussian signal. In that case it can be recovered up to indeterminacies: scaling by a constant and delay by a constant i.e.

$$\hat{s}(n) = cs(n-\Delta) \quad c \in \mathbb{C},\ \Delta \in \mathbb{N}$$

# ICA for convolutive mixtures

Statistically independent but temporally dependent source signals (audio signals for example) can be recovered blindly only up to permutation and scalar filtering indeterminacies. That is in strong contrast to instantaneous mixtures that for the same scenario allow recovery of the sources up to the scaling by a constant indeterminacy.

The scalar filtering indeterminacy is a consequence of a structure of temporally dependent time series that can be modeled as a convolution of a filter $\boldsymbol{h}_i$ with an i.i.d. driving sequence $\varepsilon$:

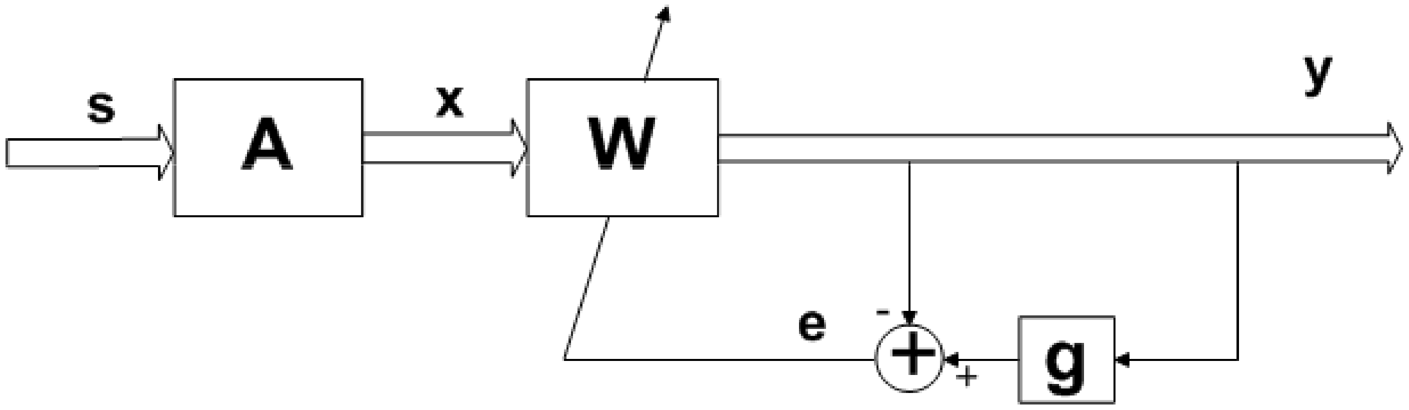
$$s_i(n) = \sum_{p=0}^{P} h_i(p)\varepsilon(n-p)$$

Thus, complete deconvolution would destroy a "color" of the signals. Thus, spatial separation between the sources is enough.

In multichannel data communication systems both separation and deconvolution are required. However, all the sources must be i.i.d. non-Gaussian signals.

# ICA for convolutive mixtures

Blind multichannel inverse modeling, equalization and separation: the input signal is unknown and there is no reference or desired signal.




In relation to instantaneous BSS problem, elements of the mixing matrix **A** in convolutive model are filters $a_{ij}$. They contain impulse responses between the $j^{th}$ input and $i^{th}$ output. We shall assume the number of inputs and outputs to be the same and equal to N.

# ICA for convolutive mixtures

**Notation and Z-transform preliminaries.** NxN mixing matrix is described as:

$$\mathbf{A}(z) = \sum_{n=-\infty}^{\infty} \mathbf{A}_n z^{-n} = \left[ a_{ij}(z) \right]$$

$$a_{ij}(z) = \sum_{n=-\infty}^{\infty} a_{ij,n} z^{-n} \quad i, j = 1 \ldots N.$$

It is assumed that each channel is stable i.e. $\sum_{n=-\infty}^{\infty} \left| a_{ij,n} \right| < \infty$.

**A**(z) is a *polynomial matrix* or *Laurent-series matrix* (a matrix whose elements are polynomials, power series or Laurent series).

$\mathbf{A}_n$ is coefficient of the matrix polynomial or matrix Laurent series (a polynomial or Laurent series whose coefficients are matrices).

# ICA for convolutive mixtures

Convolutive model in *z*-transform domain yields:

$$\mathbf{x}(z) = \mathbf{A}(z)\mathbf{s}(z)$$

where source, measured and recovered signals are described by two-sided *z*-transform:

$$s_i(z) = \sum_{n=-\infty}^{\infty} s_{i,n} z^{-n}$$

$$x_i(z) = \sum_{n=-\infty}^{\infty} x_{i,n} z^{-n}$$

$$y_i(z) = \sum_{n=-\infty}^{\infty} y_{i,n} z^{-n} \quad i = 1 \ldots N$$

The inverse system is described with:

$$\mathbf{W}(z) = \sum_{n=-\infty}^{\infty} \mathbf{W}_n z^{-n} = \left[ w_{ij}(z) \right]$$

$$w_{ij}(z) = \sum_{n=-\infty}^{\infty} w_{ij,n} z^{-n} \quad i, j = 1 \ldots N$$

# ICA for convolutive mixtures

Real channels are causal and have finite order *L*

$$\mathbf{A}(z)=\sum_{n=0}^{L}\mathbf{A}_n z^{-n}=\left[a_{ij}(z)\right]$$

Channel order *L* has to be estimated and is related to the maximal delay $\tau_{max}$ that can occur in the multipath scenario:

$$L\geq\tau_{\max}F_s$$

where $F_s$ represents sampling frequency. However, noncausal representation is necessary to model inverse of the non-minimum phase (NMP) channels:

$$\mathbf{W}(z)=\sum_{n=-L}^{L}\mathbf{W}_n z^{-n}=\left[w_{ij}(z)\right]$$

$$w_{ij}(z)=\sum_{n=-L}^{L}w_{ij,n}z^{-n}\quad i,j=1...N.$$

# ICA for convolutive mixtures

Reconstructed signals are obtained as:

$$y_i(t) = \sum_{j=1}^{N} \sum_{l=-L}^{L} w_{ij,l} x_j(t-l) \quad i = 1...N$$

Noncausal implementation of the inverse systems requires that measured signals **are known ahead in time.** Because this can not be realized a delay line of *L* samples is introduced in order to realize noncausal implementation:

$$y_i(t-L) = \sum_{j=1}^{N} \sum_{l=-L}^{L} w_{ij,l} x_j(t-L-l) \quad i = 1...N$$

This implies delay of *L* samples independently on whether adaptive or block-adaptive implementations are used.

# ICA for convolutive mixtures

**Why stable inverse of NMP system requires non-causal implementation?**

Consider a first order transfer function:

$$w_{ij}(z)=\frac{1}{1-az^{-1}}$$

for some real $a$. $w_{ij}(z)$ is consider to be inverse of the direct filter $a_{ij}(z)=1-az^{-1}$. Region of convergence (ROC) is given with:

$$|z|>|a|$$

Let us assume that $|a|<1$ in which case poles of $w_{ij}(z)$ lies inside the unit circle for $z=exp(j\omega)$ and $w_{ij}(z)$ can be represented by causal (one-sided) $z$-transform:

$$w_{ij}(z)=\frac{1}{1-az^{-1}}=\sum_{n=0}^{\infty}a^n z^{-n}$$

# ICA for convolutive mixtures

Let us now assume that $|a|>1$ in which case poles of $w_{ij}(z)$ lies outside the unit circle for $z=exp(j\omega)$. In this case the ROC becomes

$$|z|<|a|$$

A sequence with $z$-transform $w_{ij}(z)=1/(1-az^{-1})$ and above ROC is given with:

$$w_{ij}(n)=-a^n u(-n-1)$$

where $u()$ represents step function. $z$-transform of $w_{ij}(n)$ can now be written as:

$$w_{ij}(z)=\sum_{n=-\infty}^{\infty} w_{ij}(n)z^{-n}=\sum_{n=-\infty}^{\infty} -a^n u(-n-1)z^{-n}=-\sum_{n=-\infty}^{-1} a^n z^{-n}$$

which represents stable $(1/|a|)<1$ but non-causal $z$-transform necessary to implement inverse of the NMP channel $a_{ij}(z)=1-az^{-1}$.

# ICA for convolutive mixtures

**Single channel blind deconvolution.**

$$y(t) = \sum_{l=-L}^{L} w_l x(t-l)$$

$$x(t) = \sum_{l=0}^{L} a_l s(t-l)$$

**Assumptions:**

A1) Source signal must be non-Gaussian. Due to the central limit theorem *x(t)* is very close to Gaussian process even if *s(t)* is non-Gaussian. Maximizing departure from Gaussianity does not make sense if source signal is Gaussian.

A2) Source signal must be independent identically distributed (i.i.d.) process (temporally white):

$$E\left[s(t)s(t-l)\right] = \sigma\delta_l$$

$$\mathbf{p}(s(t), s(t-l)) = p(s(t))\,p(s(t-l)) \vee l > 0$$

$$p_s(t_1) = p_s(t_2)$$

# ICA for convolutive mixtures

Single channel blind deconvolution problem as instantaneous ICA problem:

$$\tilde{\mathbf{x}} \cong \mathbf{A}\tilde{\mathbf{s}}$$

$$\tilde{\mathbf{s}}(t) = \left[ s(t)\ s(t-1)\ \dots\ s(t-2L+1) \right]^T$$

$$\tilde{\mathbf{x}}(t) = \left[ x(t)\ x(t-1)\ \dots\ x(t-L+1) \right]^T$$

$$\mathbf{A} = \begin{bmatrix} a_1\, a_2 \dots\dots a_L \\ 0\, a_1\, a_2 \dots a_{L-1} \\ \dots\dots\dots\dots\dots\dots \\ 0 \dots\dots 0\ \ a_1 \end{bmatrix}$$

ICA independence assumption implies:

$$E\left[ s(t)s(t-1)\dots s(t-L) \right] = E[s(t)]E[s(t-1)]\dots E[s(t-L)]$$

$$\mathbf{p}(s(t), s(t-1), \dots, s(t-L)) = p(s(t))\,p(s(t-1))\dots p(s(t-L))$$

that is equivalent to i.i.d. assumption.

# ICA for convolutive mixtures

**Why colored signals can not be completely deconvolved?**

Z-transform of the white (i.i.d.) source signal:

$$S(z) = \sigma_s^2 z^{-\Delta_s}$$

Z-transform of the colored source signal:

$$S(z) = \sigma_s^2 D(z) z^{-\Delta_s}$$

$$D(z) = \sum_p d_p z^{-p}$$

Z-transform of the reconstructed signal:

$$Y(z) = W(z)A(z)S(z)$$

# ICA for convolutive mixtures

Minimization of statistical independence between the sources (spatial separation) yields sources:

$$\hat{S}_i(z) = P_i(z)S_i(z) = \sigma_{s_i}^2 P_i(z)D_i(z)z^{-\Delta_{s_i}} \quad i = 1,\ldots,N$$

That is because if : $\{s_i(n)\}_{i=1}^N$ are independent so are $\{\hat{s}_i(n)\}_{i=1}^N$ .

Thus, exploiting statistical independence assumption only, the non-i.i.d. (color) sources can be recovered up to the scalar filtering indeterminacy only.

To, possibly, remove $P_i(z)$ further processing of $\{\hat{s}_i(n)\}_{i=1}^N$ is necessary.

Since, $\{\hat{s}_i(n)\}_{i=1}^N$ are statistically independent the only redundancy that is left is temporal dependence within each $\{\hat{s}_i(n)\}_{i=1}^N$.

# ICA for convolutive mixtures

Goal in single channel blind deconvolution:

$$Y(z) \cong S(z)z^{-\Delta}$$

Unsupervised learning criteria are based on the maximization of independence between samples of the sequence *y(t)*:

$$f_y(y(t), y(t-1),..., y(t-m+1)) = \prod_{i=0}^{m-1} f(y(t-i))$$

this will yield:

$$E\left[y(t-i)y(t-j)\right] = \sigma_y^2 \delta_{ij}$$

which implies:

$$Y(z) = \sigma_y^2 z^{-\Delta_y}$$

which further implies that: $P_i(z)D_i(z) \rightarrow \sigma_y^2 z^{-\Delta_{y_i}} \quad i = 1,..., N$

i.e. colored signals can not be recovered in completely blind scenario.

# ICA for convolutive mixtures

**Multichannel blind deconvolution.** The same assumptions extend to multichannel blind deconvolution (MBD=BSS+SBD):

A1) all source signals must be non-Gaussian.
A2) all source signals must be statistically independent and i.i.d. processes:

$$E\left[s_i(t-r)s_j(t-q)\right]=\sigma^2\delta_{ij}\delta_{rq}$$

$$p\left(s_i(t-r)s_j(t-q)\right)=p\left(s_i(t-r)\right)p\left(s_j(t-q)\right)\ \forall r\neq q$$

General solution of the blind source separation problem is given with:

$$\mathbf{G}(z)=\mathbf{W}(z)\mathbf{A}(z)=\mathbf{P\Lambda D}(z)$$

Where **P** is general permutation matrix, $\Lambda$ is diagonal scaling matrix and

$$\mathbf{D}(z)=diag\left\{d_1z^{-\Delta_1}...d_Nz^{-\Delta_N}\right\}$$

Objective of the BSS in the most general case is to recover possibly scaled, reordered and filtered estimates of the unknown source signals.

# ICA for convolutive mixtures

- Multichannel blind separation and deconvolution in time domain

  (+) On-line formulation is realizable.

  (-) Slow convergence.

  (-) Colored signals are whitened if feedforward architecture is used.

  (-) Whitening can be avoided with feedback architecture but this prevents implementation of the non-causal inverse filters required to implement stable inverse of the NMP mixing channels.

- Multichannel blind separation and deconvolution in frequency domain

  (+) Convergence is faster due to the fact that frequency bins are orthogonal.

  (+) Condition for non-causal implementation is satisfied naturally through block filtering implementation.

  (-) Only block-adaptive but not truly adaptive implementation can be achieved.

  (-) Permutation on the frequency bin levels causes serious difficulties when signals have to be transformed back in time domain.

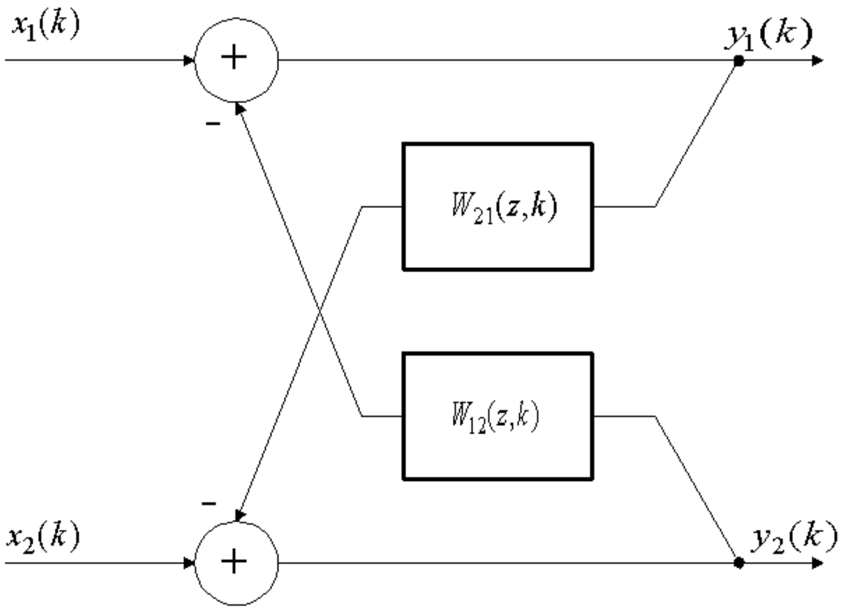
  (-) Computationally more complex to implement.

# ICA for convolutive mixtures

**Time domain approach with recurrent (feedback) architecture.**

(+) Whitening effect is eliminated due to the fact that signals $y_i(t)$ have to have temporal structure in order to cancel appropriate source signal $s_j(t)$ in the mixture $x_n(t)$.

(-) It is not possible to realize non-causal implementation necessary to invert non-minimum phase mixture channels.

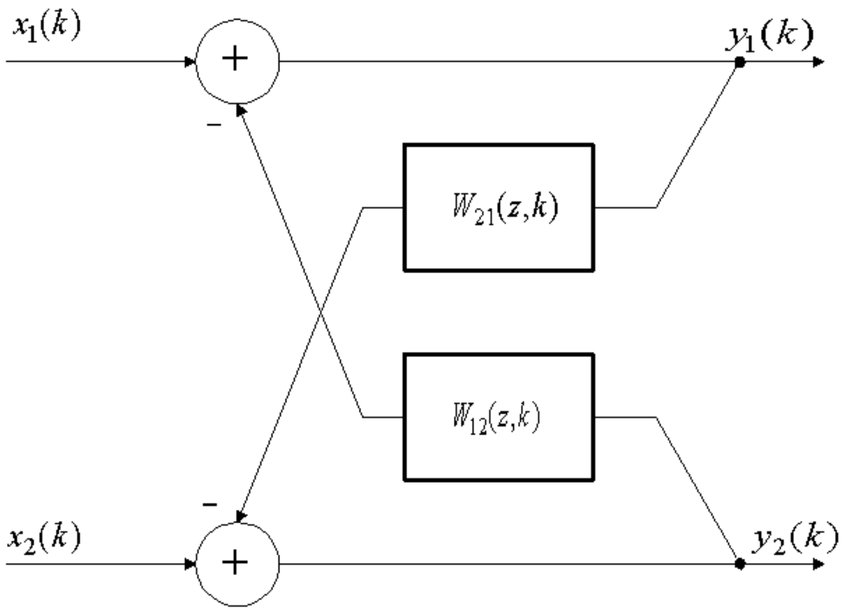



$$y_1(k) = x_1(k) - \sum_{l=1}^{L} w_{12}(l) y_2(k-l)$$

$$y_2(k) = x_2(k) - \sum_{l=1}^{L} w_{21}(l) y_1(k-l)$$

# ICA for convolutive mixtures

**Time domain approach with recurrent (feedback) architecture.**
Asymptotic solutions for unmixing filters.




$$W_{12}(z) = A_{12}(z)A_{22}^{-1}(z)$$

$$W_{21}(z) = A_{21}(z)A_{11}^{-1}(z)$$

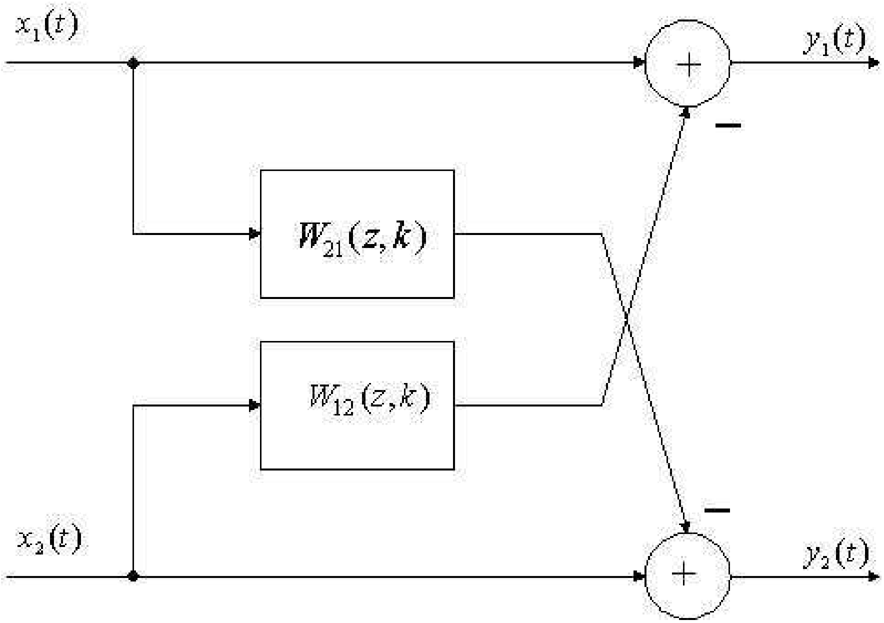
or

$$W_{12}(z) = A_{11}(z)A_{21}^{-1}(z)$$

$$W_{21}(z) = A_{22}(z)A_{12}^{-1}(z)$$

# ICA for convolutive mixtures

**Time domain approach with feedforward architecture.**
(+) Non-causal realization necessary to invert non-minimum phase mixtures is easily implemented by pure delay line.
**(-)** Direct filters set to unity only partially eliminate the whitening effect.

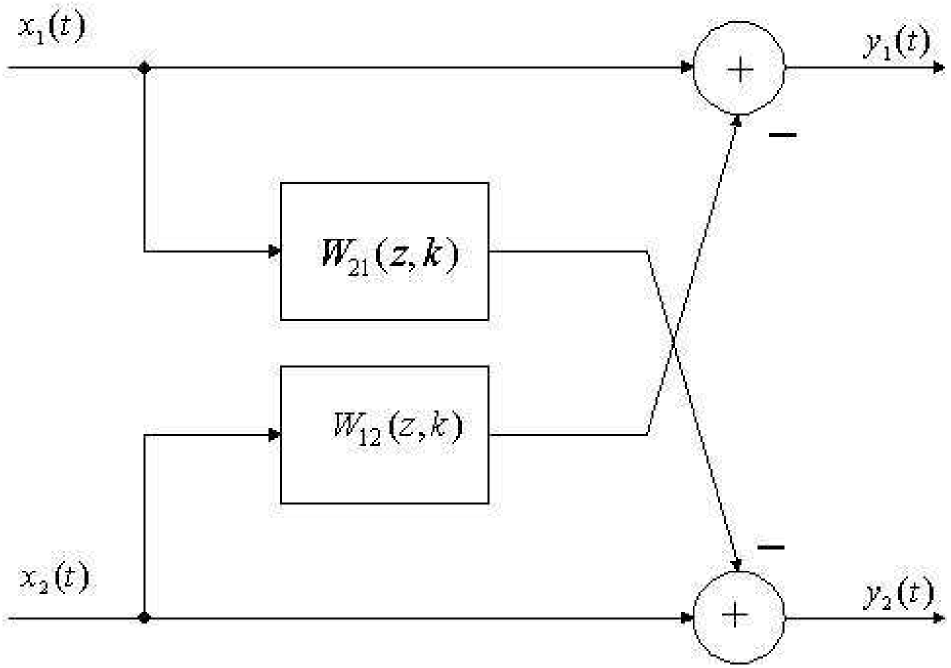



$$y_1(k) = x_1(k) - \sum_{l=-L}^{L} w_{12}(l) x_2(k-l)$$

$$y_2(k) = x_2(k) - \sum_{l=-L}^{L} w_{21}(l) x_1(k-l)$$

# ICA for convolutive mixtures

**Time domain approach with feedforward architecture.** Asymptotic solutions for unmixing filters.




$$W_{12}(z) = -A_{12}(z)A_{22}^{-1}(z)$$

$$W_{21}(z) = -A_{21}(z)A_{11}^{-1}(z)$$

or

$$W_{12}(z) = -A_{11}(z)A_{21}^{-1}(z)$$

$$W_{21}(z) = -A_{22}(z)A_{12}^{-1}(z)$$

# ICA for convolutive mixtures

**Contrast function** for MBD problems are in a majority of cases derived or can be related to maximum-likelihood formulation of the MBD problem[5,6].

$$J_{ML}(\mathbf{W}) = \int_{\mathbf{x}} f_{\mathbf{x}}(\mathbf{x}) \log \hat{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{W}) \mathbf{dx} = E\left\{\log \hat{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{W})\right\}$$

that amounts to minimizing spatial and temporal independence. Noting that

$$\hat{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{W}) = \left[\hat{f}_{\mathbf{x}}(\mathbf{x}, \mathbf{W}) / f_{\mathbf{x}}(\mathbf{x})\right] f_{\mathbf{x}}(\mathbf{x})$$

likelihood contrast function can be written as

$$J_{ML}(\mathbf{W}) = -D\left(f_{\mathbf{x}} \middle\| \hat{f}_{\mathbf{x}}\right) - H(f_{\mathbf{x}})$$

where *D*() represents Kullback-Leibler distance.

6D.L. Donoho, "On minimum entropy deconvolution," in D.F. Findley, ed., *Applied Time Series Analysis II,* Academic Press, pp.565-608, 1981.

# ICA for convolutive mixtures

Learning rule for **W** is obtained as:

$$\Delta \mathbf{W} \propto \frac{\partial J_{ML}(\mathbf{W})}{\partial \mathbf{W}} = -\frac{\partial D(f_{\mathbf{x}} \| \hat{f}_{\mathbf{x}})}{\partial \mathbf{W}}$$

which means that maximization of likelihood is equivalent to minimization of distance between true unknown and model pdf's for a set of measurement.

When MBD learning rules are derived by maximizing likelihood function they will contain score functions as nonlinear functions:

$$\varphi_i(y_i) = -\frac{1}{f_{s_i}(y_i)} \frac{df_{s_i}(y_i)}{dy_i}$$

# ICA for convolutive mixtures

As shown before parameterized form of the score functions can be derived from generalized Gaussian distribution:

$$\varphi_i(y_i) = sign(y_i)\left|y_i\right|^{\alpha_i - 1}$$

With the single parameter $\alpha_i$ (called Gaussian exponent) super-Gaussian distributions ($\alpha_i$ <2) and sub-Gaussian distributions ($\alpha_i$ >2) could be modeled.

If MBD is applied in communication environment $\alpha_i=3$ is good choice yielding:

$$\varphi_i(y_i) = sign(y_i)\left|y_i\right|^{2}$$

If MBD is applied on audio signals $\alpha_i=1$ is good choice yielding:

$$\varphi_i(y_i) = sign(y_i)$$

# ICA for convolutive mixtures

**On-line (time domain) *Infomax* or maximum likelihood ICA algorithm for blind separation of convolved minimum phase mixtures** with feedback architecture. Colored signals such as speech could be separated.

$$\Delta w_{ij}(t,l) \cong \frac{\partial H(\varphi(\mathbf{y}))}{\partial w_{ij}(l)} = \varphi_i(y_i)y_j(t-l)$$

$$w_{ij}(t+1,l) = w_{ij}(t,l) + \mu\Delta w_{ij}(t,l)$$

$$y_i(t) = x_i(t) - \sum_{\substack{j\neq i\\ j=1}}^{N}\sum_{l=1}^{L} w_{ij}(t,l)y_j(t-l)$$

where *i,j* denote signal indices, *t* denotes iteration index, *l* denotes coefficient index and $\mu$ represents learning gain.

# ICA for convolutive mixtures

**Frequency domain approach to MBD.**

$$\mathbf{X} = \mathrm{FFT}[\mathbf{x}]$$

$$\mathbf{W}_k(l+1) = \mathbf{W}_k(l) + \left[\mathbf{I} - \mathbf{\Phi}\left(\mathbf{Y}_k\right)\mathbf{Y}_k^H\right]\mathbf{W}_k(l)$$

$$\mathbf{Y}_k = \mathbf{W}_k\mathbf{X}_k$$

$$\mathbf{y} = \mathrm{IFFT}\left[\mathbf{Y}\right]$$

Where *k* denotes frequency bin index and *l* denotes iteration index. Permutation indeterminacy is a serious problem if MBD is implemented completely in frequency domain .

$$\left(\mathbf{W}_{k_1}\mathbf{A}_{k_1} = \mathbf{P}_{k_1}\mathbf{\Lambda}_{k_1}\right) \neq \left(\mathbf{W}_{k_2}\mathbf{A}_{k_2} = \mathbf{P}_{k_2}\mathbf{\Lambda}_{k_2}\right)$$

Components on the same positions at different frequency bins do not belong to the same signal. Nonlinear function in frequency domain can be used as [7]

$$\Phi_k(Y_k) = \tanh\left(\eta\left|Y_k\right|\right)e^{j\arg(Y_k)}$$

7. H. Sawada, R. Mukai, S. Araki, S. Makino, "Polar Coordinate based Nonlinear Function for Frequency-Domain Blind Source Separation," IEICE Trans. Fundamentals, Vol. E86-A, No. 3, March 2003.

# ICA for convolutive mixtures

**Mixed implementation in time and frequency domain.** Trade-off solution is to execute filtering in frequency domain and perform statistical independence test (that causes permutation indeterminacy) in time domain[8,9]

$$\mathbf{X} = \mathrm{FFT}[\mathbf{x}]$$

$$\Phi_i\left(\mathbf{Y}_i\right) = \mathrm{FFT}\left[\varphi(\mathbf{y}_i)\right]$$

$$\mathbf{W}_k(l+1) = \mathbf{W}_k(l) + \mu\left[\mathbf{I} - \mathbf{\Phi}\left(\mathbf{Y}_k\right)\mathbf{Y}_k^H\right]\mathbf{W}_k(l)$$

$$\mathbf{Y}_k = \mathbf{W}_k\mathbf{X}_k$$

$$\mathbf{y}_i = \mathrm{IFFT}\left(\mathbf{Y}_i\right)$$

Where *k* denotes frequency bin index, *l* denotes iteration index, *i* denotes signal index and $\mu$ is small learning gain.

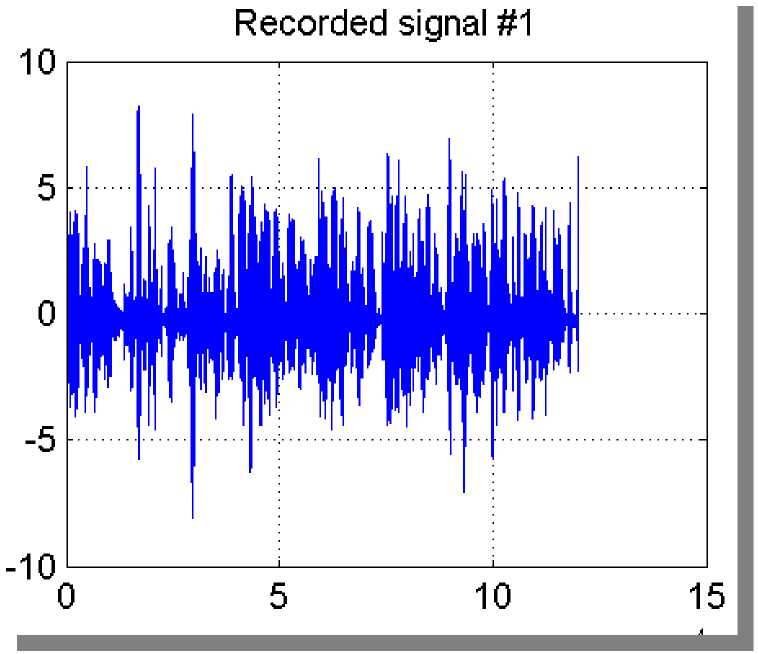
8 I.Kopriva, H. Szu, A.Persin, Optics Comm., Vol. 203 (3-6) pp. 197-211, 2002.
9. A. D. Back, A.C. Tsoi, Proc. of the 1994 IEEE Workshop – Neural Networks for Signal Processing IV, p.565, ed. J. Vlontzos, J.N. Hwang, E. Wilson,

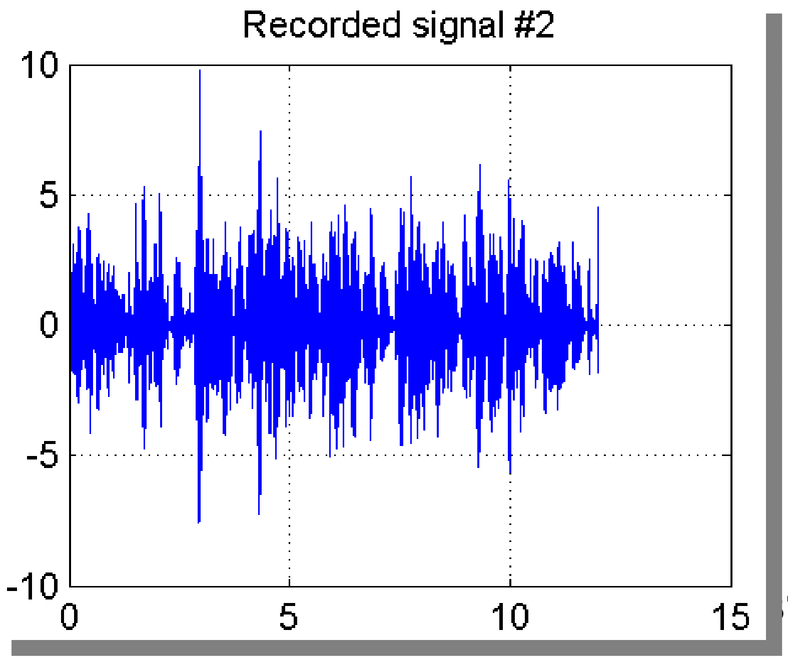
# Applications of convolutive ICA

**Speech separation in reverberant acoustic environment.** Two recorded signals were downloaded from Russel Lamberts' home page:

http://home.socal.rr.com/russdsp/ .

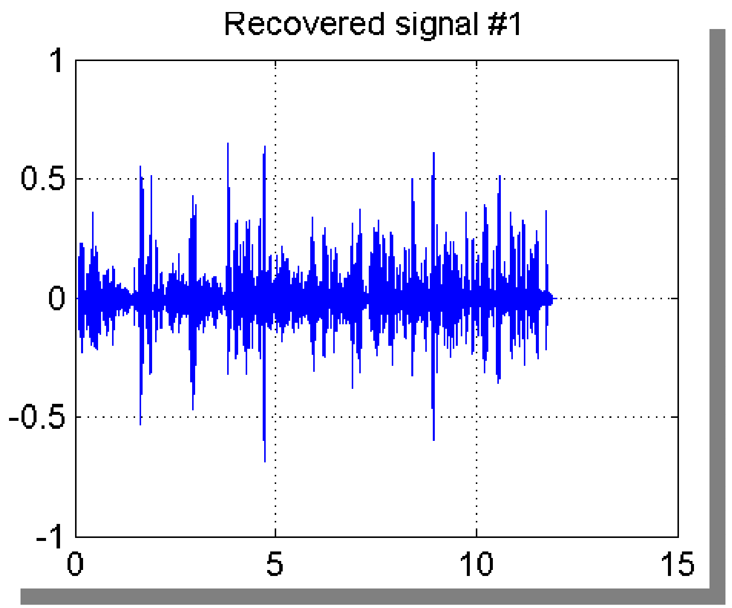
Signals were sampled with 8kHz and contain male and female speakers talking simultaneously for 12 seconds.

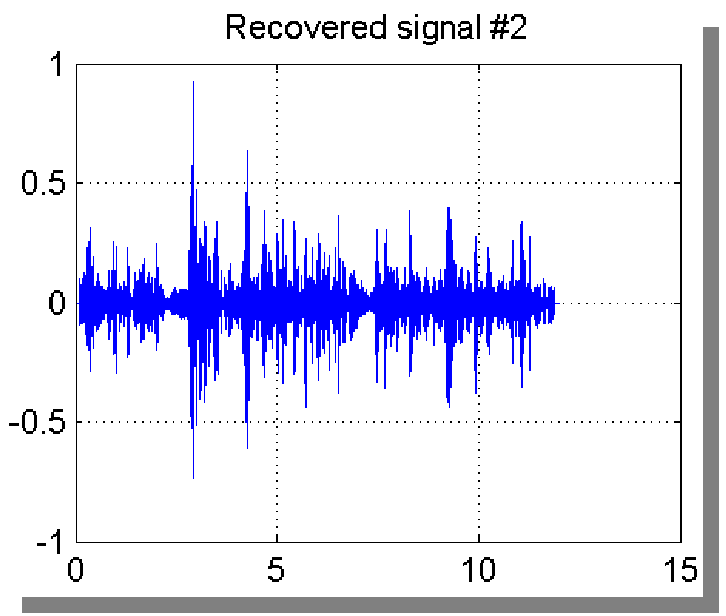


# Applications of convolutive ICA

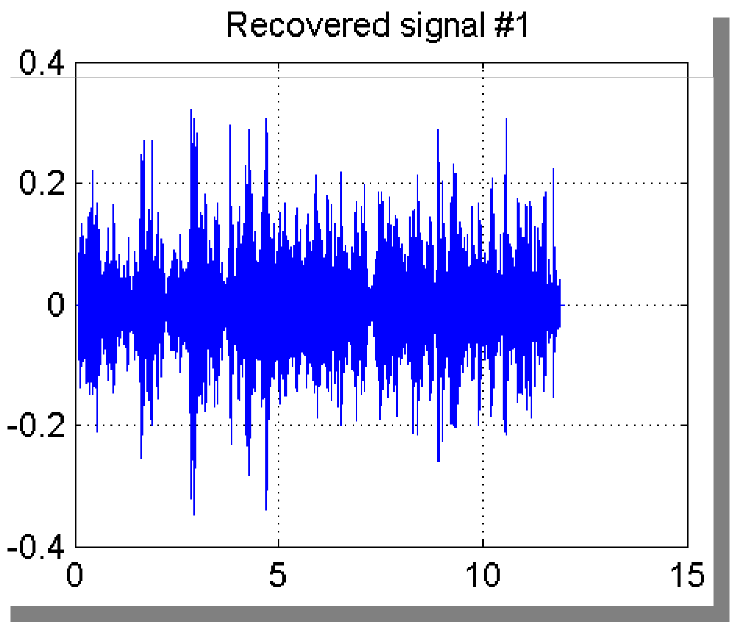
Parameters of the separation process were filter length *L*, Gaussian exponent $\alpha_l$ and learning gain $\mu$.. At sampling frequency 8kHz and filter length *L*=1024 a relative delay of 64ms could be approximated. With speed of sound in the air of 330 $ms^{-1}$ this corresponds with path length difference of 21m. The following signals were recovered with *L*=1024 , $\alpha_i$ =1.0 and $\mu$=0.005.

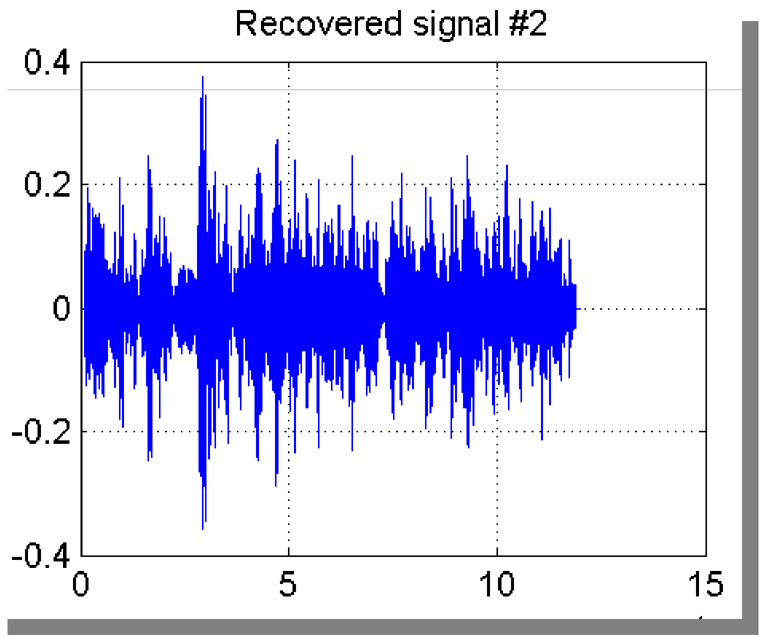


# Applications of convolutive ICA

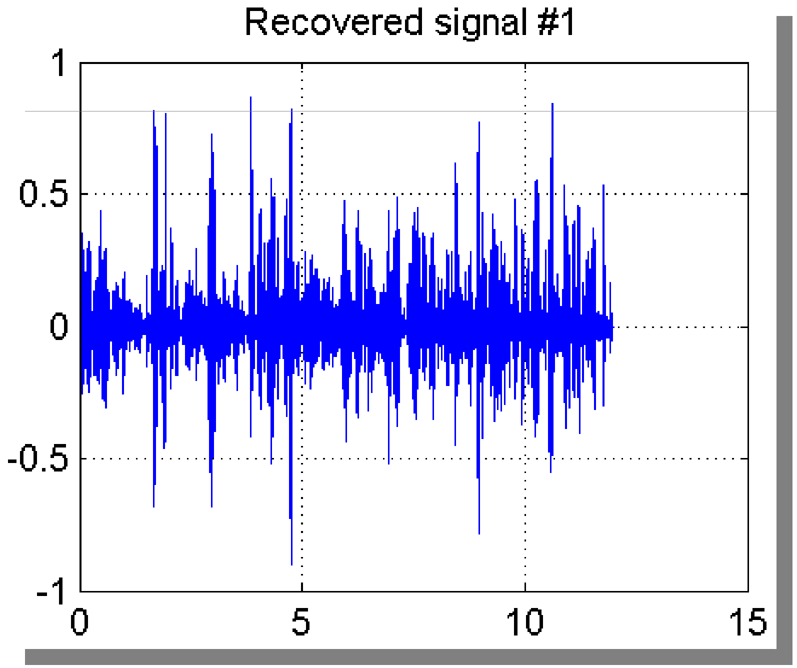
Following signals were recovered with $L$=1024 , $\alpha_i$ =3.0 and $\mu$=0.005. Choice of $\alpha_i$ =3.0 corresponds with sub-Gaussian distributions and is wrong for speech signals.

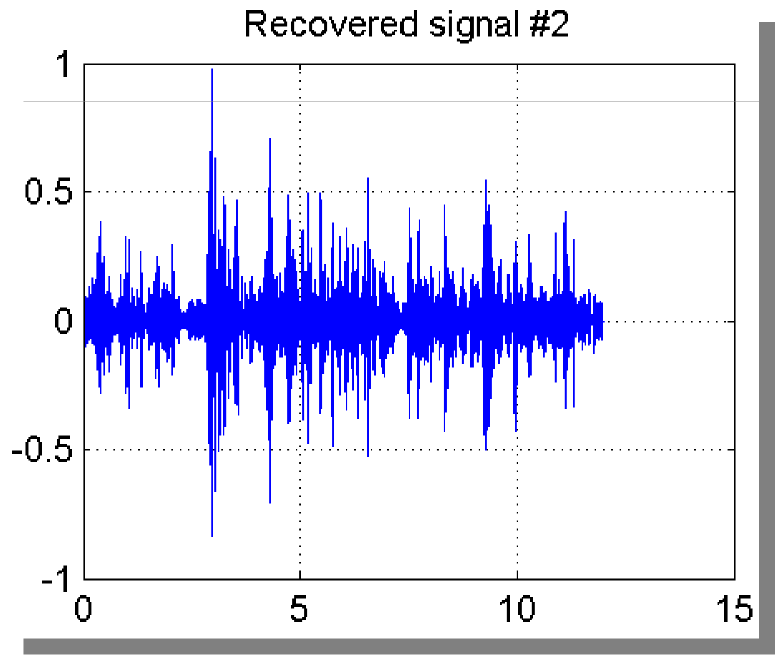


# Applications of convolutive ICA

Following signals were recovered with $L$=256, $\alpha_i$ =1.0 and $\mu$=0.0013. Choice of $L$=256 will model relative path difference up to 5m.



# ICA for convolutive mixtures

If source signals are white (i.i.d.) **feedforward architecture can be employed in time domain** to realize on line non-causal implementation with natural gradient capable to approximate inverse of the NMP channels[3,10].

$$\mathbf{y}(t) = \sum_{p=0}^{L} \mathbf{W}_p(t)\mathbf{x}(t-p)$$

$$\mathbf{u}(t) = \sum_{p=0}^{L} \mathbf{W}_{L-p}^{H}(t)\mathbf{y}(t-p)$$

$$\mathbf{W}_p(t+1) = \mathbf{W}_p(t) + \eta(t)\left(\mathbf{W}_p(t) - \varphi(\mathbf{y}(t-L))\mathbf{u}^H(t-p)\right) \quad p = 0,\ldots,L$$

where *g()* is functional inverse of $\varphi$ such that:

$$\varphi_i(g_i(y_i)) = y_i$$

And *L* is filter length, *t* is time index, *p* is index of the coefficient of the matrix polynomial and $\eta$ is small learning gain.



10. S. Amari, S.C.Douglas, A. Cihocki and H.H. Yang, "Multichannel Blind Deconvolution and Equalization Using the Natural Gradient," IEEE International Workshop on Wireless Communication, Paris 1997, pp. 101-104.

# Applications of convolutive ICA

**Multichannel blind equalization** of 3x3 systems with 3 i.i.d. source signals (QAM)[10] with the NMP channels.

$$\mathbf{x}(k) = \sum_{i=1} \mathbf{A}_i \mathbf{x}(k-i) + \sum_{j=0} \mathbf{B}_j s(k-j), \qquad (31)$$

$$\mathbf{A}_1 = \begin{bmatrix} -0.56+0.35j & 0.34+0.08j & -0.14-0.40j \\ -0.28+0.08j & 0.18+0.43j & -0.66-0.15j \\ -0.08-0.27j & -0.40+0.15j & 0.16-0.36j \end{bmatrix} \qquad (32)$$

$$\mathbf{A}_2 = \begin{bmatrix} 0.04+0.03j & 0.02+0.16j & 0.03-0.09j \\ 0.08-0.05j & 0.04+0.08j & -0.01j \\ 0.03 & 0.06+0.03j & 0.06+0.01j \end{bmatrix} \qquad (33)$$

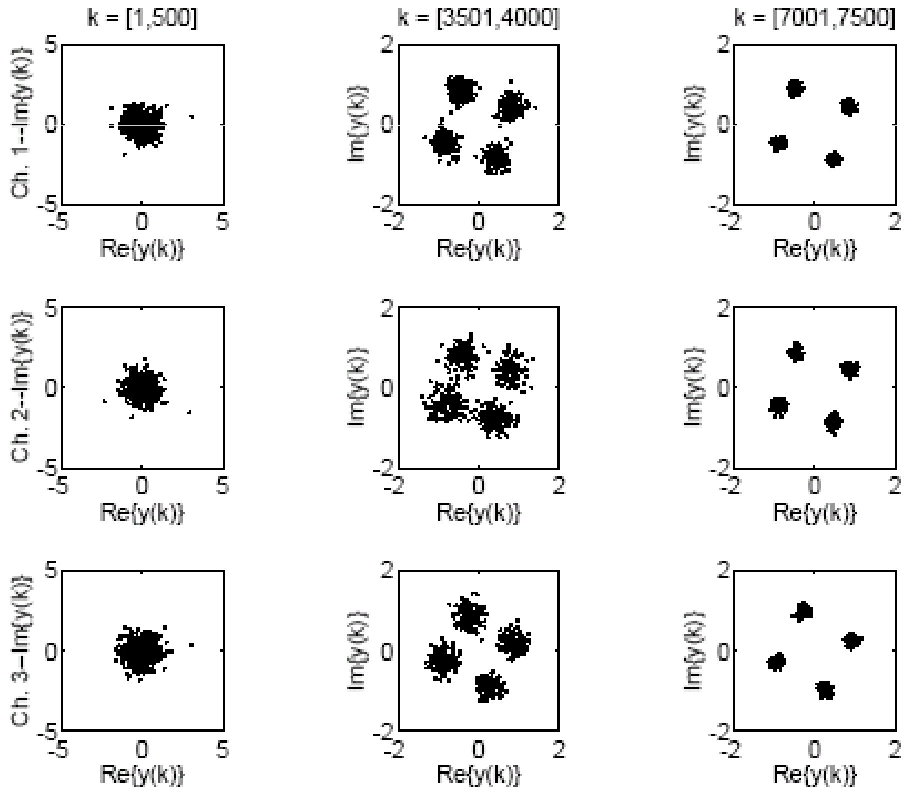
$$\mathbf{B}_0 = \begin{bmatrix} 0.02+0.09j & 0.07+0.01j & 0.05+0.07j \\ 0.08j & 0.09+0.07j & 0.08+0.09j \\ 0.07+0.05j & 0.04+0.04j & 0.08j \end{bmatrix} \qquad (34)$$

$$\mathbf{B}_1 = \begin{bmatrix} 0.1+0.3j & 0.3j & 0.4+j \\ 0.5 & 0.4+0.6j & 0.7+0.4j \\ 0.7+0.7j & 0.1+0.8j & 0.6+0.2j \end{bmatrix}. \qquad (35)$$

# Applications of convolutive ICA

Output constellations for blind equalizer are shown for all three restored sources for time intervals $1 \leq t \leq 500$, $3501 \leq t \leq 4000$ and $7001 \leq t \leq 7500$.

$y_i(t) \approx s_i(t-4)$.



# Applications of convolutive ICA

Output constellations are shown multichannel LMS equalizer trained with $d_i(t)=s_i(t-4)$. Constellations are shown on intervals $1 \le t \le 500$, $3501 \le t \le 4000$ and $7001 \le t \le 7500$.